

**Sample Question Paper - 1**  
**CLASS: XII**  
**Session: 2021-22**  
**Mathematics (Code-041)**  
**Term - 1**

**Time Allowed: 1 hour and 30 minutes**

**Maximum Marks: 40**

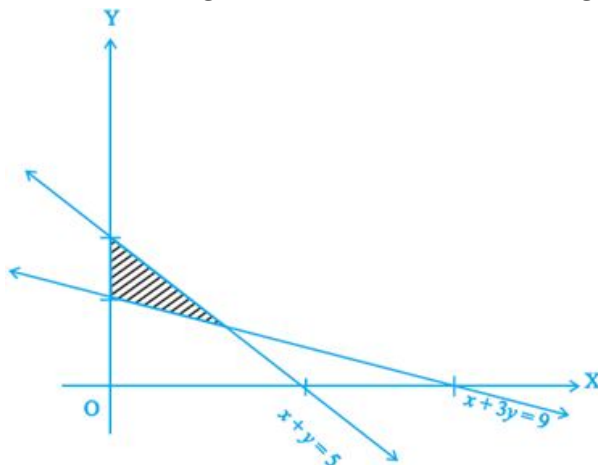
**General Instructions:**

1. This question paper contains three sections - A, B and C. Each part is compulsory.
2. Section - A has 20 MCQs, attempt any 16 out of 20.
3. Section - B has 20 MCQs, attempt any 16 out of 20
4. Section - C has 10 MCQs, attempt any 8 out of 10.
5. There is no negative marking.
6. All questions carry equal marks.

**Section A**

**Attempt any 16 questions**

1. The relation R defined on the set  $A = \{1, 2, 3, 4, 5\}$  by  $R = \{(a, b) : |a^2 - b^2| < 16\}$ , is given by **[1]**
  - a)  $\{(2, 2), (3, 2), (4, 2), (2, 4)\}$
  - b)  $R = \{(1, 1), (2, 1), (3, 1), (4, 1), (2, 3), (2, 2), (3, 2), (4, 2), (2, 4), (3, 3), (5, 4), (3, 4)\}$
  - c) none of these
  - d)  $\{(3, 3), (4, 3), (5, 4), (3, 4)\}$
2. The feasible region for a LPP is shown in Figure. Find the maximum value of  $Z = 11x + 7y$ . **[1]**



- a) 49
  - b) 48
  - c) 50
  - d) 47
3. Derivative of  $\cos(\sin x)$  w.r.t.  $\sin x$  is **[1]**
    - a)  $-\sin(\sin x)\cos x$
    - b)  $\sec^2\left(\frac{x}{\sqrt{x^2+1}}\right)$
    - c)  $-\sin(\sin x)$
    - d) None of these

4. If  $2 \begin{bmatrix} 3 & 4 \\ 5 & x \end{bmatrix} + \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$  then [1]

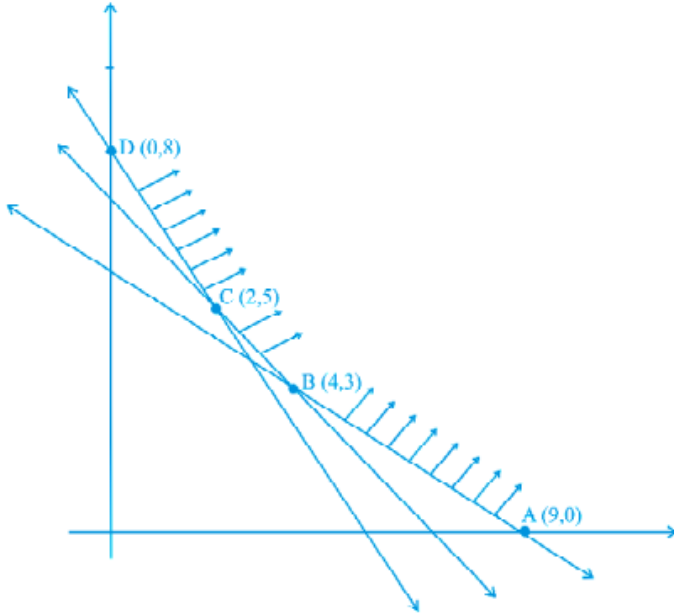
a)  $(x = 2, y = 8)$

b)  $(x = 3, y = -6)$

c)  $(x = -3, y = 6)$

d)  $(x = 2, y = -8)$

5. Feasible region (shaded) for a LPP is shown in the Figure. Minimum of  $Z = 4x + 3y$  occurs at the point [1]



a)  $(4, 3)$

b)  $(9, 0)$

c)  $(0, 8)$

d)  $(2, 5)$

6. If  $y = \tan^{-1}(\sec x + \tan x)$  then  $\frac{dy}{dx} = ?$  [1]

a) None of these

b)  $\frac{1}{2}$

c) 1

d)  $\frac{-1}{2}$

7. For an invertible square matrix of order 3 with real entries  $A^{-1} = A^2$ , then  $\det. A =$  [1]

a)  $\frac{1}{3}$

b) 3

c) None of these

d) 1

8. The slope of the tangent to the curve  $x = a(\cos \theta + \theta \sin \theta)$ ,  $y = a(\sin \theta - \theta \cos \theta)$  at any point 'θ' is [1]

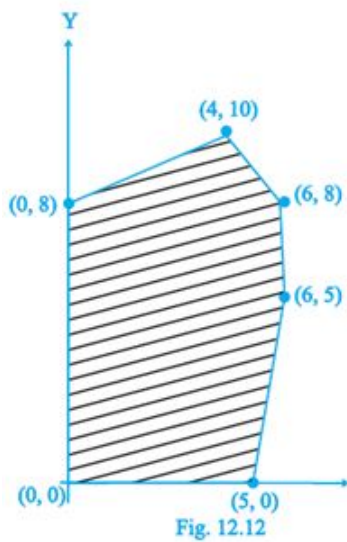
a)  $\tan \theta$

b)  $-\cot \theta$

c)  $-\tan \theta$

d)  $\cot \theta$

9. The feasible solution for an LPP is shown in Figure. Let  $Z = 3x - 4y$  be the objective function. Maximum value of  $Z$  occurs at [1]



- a) (6, 8) b) (6, 5)  
 c) (4, 10) d) (5, 0)
10. If  $A = \begin{bmatrix} 2 & -1 & 3 \\ -4 & 5 & 1 \end{bmatrix}$  and  $B = \begin{bmatrix} 2 & 3 \\ 4 & -2 \\ 1 & 5 \end{bmatrix}$  then [1]
- a) only BA is defined b) only AB is defined  
 c) AB and BA both are not defined d) AB and BA both are defined
11. The function  $f(x) = \begin{cases} \frac{\sin x}{x} + \cos x, & \text{if } x \neq 0 \\ k, & \text{if } x = 0 \end{cases}$  is continuous at  $x = 0$ , then the value of k is [1]
- a) 1 b) 3  
 c) 1.5 d) 2
12. Which of the following statements is correct? [1]
- a) A LPP admits unique optimal solution b) Every LPP admits an optimal solution  
 c) If an LPP admits two optimal solutions it has an infinite number of optimal solutions d) The set of all feasible solutions of a LPP is not a convex set
13. If the function  $f(x) = kx^3 - 9x^2 + 9x + 3$  is monotonically increasing in every interval, then [1]
- a)  $k > 3$  b)  $k < 3$   
 c)  $k \geq 3$  d)  $k \leq 3$
14. If  $f(x) = \begin{cases} \frac{1}{1+e^{1/x}}, & x \neq 0 \\ 0, & x = 0 \end{cases}$  then f(x) is [1]
- a) none of these b) differentiable but not continuous at  $x = 0$   
 c) continuous but not differentiable at  $x = 0$  d) continuous as well as differentiable at  $x = 0$















# Solution

## Section A

1. (c) none of these

**Explanation:** Given set  $A = \{1, 2, 3, 4, 5\}$  and relation  $R = \{(a, b) : |a^2 - b^2| < 16\}$

According to the condition  $|a^2 - b^2| < 16$ :

$\Rightarrow R = \{(1, 1), (1, 2), (2, 1), (1, 3), (3, 1), (1, 4), (4, 1), (2, 3), (2, 2), (3, 2), (4, 2), (2, 4), (3, 3), (4, 3), (5, 4), (3, 4), (4, 4), (5, 5)\}$ . Which is the required solution.

2. (d) 47

**Explanation:**

Corner points	$Z = 11x + 7y$
(0, 5)	35
(0, 3)	21
(3, 2)	47

The maximum value is 47

3. (c)  $-\sin(\sin x)$

**Explanation:** Let  $y = \cos(\sin x)$ ,  $z = \sin x$ , then,  $\frac{dy}{dz} = \frac{\frac{dy}{dx}}{\frac{dz}{dx}} = \frac{-\sin(\sin x) \cos x}{\cos x} = -\sin(\sin x)$

4. (d)  $(x = 2, y = -8)$

**Explanation:**  $2 \begin{pmatrix} 3 & 4 \\ 5 & x \end{pmatrix} + \begin{pmatrix} 1 & y \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 7 & 0 \\ 10 & 5 \end{pmatrix}$

To solve this problem we will use the comparison that is we will use that all the elements of L.H.S. are equal to R.H.S.

$$= \begin{pmatrix} 6 & 8 \\ 10 & 2x \end{pmatrix} + \begin{pmatrix} 1 & y \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 7 & 8 + y \\ 10 & 2x + 1 \end{pmatrix}$$

Comparing with R.H.S.

$$8 + y = 0$$

$$y = -8$$

$$2x + 1 = 5$$

$$2x = 4$$

$$x = 2$$

5. (d) (2, 5)

**Explanation:**  $Z = 4x + 3y$

1.  $(0, 8) = 24$

2.  $(2, 5) = 8 + 15 = 23$

3.  $(4, 3) = 16 + 9 = 25$

4.  $(9, 0) = 36 + 0 = 36$

The correct answer is (2, 5) as it gives the minimum value.

6. (b)  $\frac{1}{2}$

**Explanation:** Given that  $y = \tan^{-1}(\sec x + \tan x)$

Hence,  $y = \tan^{-1}\left(\frac{1 + \sin x}{\cos x}\right)$

Using  $\cos x = \cos^2 \frac{x}{2} - \sin^2 \frac{x}{2}$ ,  $\sin x = 2 \sin \frac{x}{2} \cos \frac{x}{2}$  and  $\cos^2 \theta + \sin^2 \theta = 1$

$$\text{Hence, } y = \tan^{-1} \left( \frac{\cos^2 \frac{x}{2} + \sin^2 \frac{x}{2} + 2 \sin \frac{x}{2} \cos \frac{x}{2}}{\cos^2 \frac{x}{2} - \sin^2 \frac{x}{2}} \right) = \tan^{-1} \left( \frac{\left( \cos \frac{x}{2} + \sin \frac{x}{2} \right)^2}{\left( \cos \frac{x}{2} - \sin \frac{x}{2} \right) \left( \cos \frac{x}{2} + \sin \frac{x}{2} \right)} \right)$$

$$\Rightarrow y = \tan^{-1} \left( \frac{\cos \frac{x}{2} + \sin \frac{x}{2}}{\cos \frac{x}{2} - \sin \frac{x}{2}} \right)$$

Dividing by  $\cos \frac{x}{2}$  in numerator and denominator, we obtain

$$y = \tan^{-1} \frac{1 + \tan \frac{x}{2}}{1 - \tan \frac{x}{2}}$$

Using  $\tan \left( \frac{\pi}{4} + x \right) = \frac{1 + \tan x}{1 - \tan x}$ , we obtain

$$y = \tan^{-1} \tan \left( \frac{\pi}{4} + \frac{x}{2} \right) = \frac{\pi}{4} + \frac{x}{2}$$

Differentiating with respect to x, we

$$\frac{dy}{dx} = \frac{1}{2}$$

7. **(d)** 1

**Explanation:**  $A^2 = I \Rightarrow A^2 A^{-1} = I A^{-1} \Rightarrow A = A^{-1}$  and it is possible only if A is an identity matrix and determinant of the identity matrix is equal to 1

8. **(a)**  $\tan \theta$

**Explanation:** Given  $x = a(\cos \theta + \theta \sin \theta)$ ,  $y = a(\sin \theta - \theta \cos \theta)$

$$\frac{dx}{d\theta} = a[-\sin \theta + \theta \cdot \cos \theta + \sin \theta] = a\theta \cos \theta, \frac{dy}{d\theta} = a[\cos \theta - (\theta \cdot -\sin \theta + \cos \theta)] = a\theta \sin \theta$$

$$\text{Slope of the tangent} = \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{a\theta \sin \theta}{a\theta \cos \theta} = \tan \theta$$

9. **(d)** (5, 0)

**Explanation:**

Corner points	Z = 3x - 4y
(0, 0)	0
(5, 0)	15
(6, 8)	-14
(6, 5)	-2
(4, 10)	-28
(0, 8)	-32

The maximum value occurs at (5, 0)

10. **(d)** AB and BA both are defined

**Explanation:** In given matrix

order of A =  $2 \times 3$

order of B =  $3 \times 2$

AB will be defined if the number of column in A is equal to the number of rows in B

$$\text{so, } (A_{2 \times 3})(B_{3 \times 2}) = AB_{2 \times 2}$$

$$\text{Similarly } (B_{3 \times 2})(A_{2 \times 3}) = BA_{3 \times 3}$$

Thus, Both AB and BA are defined.

11. **(d)** 2

**Explanation:** Since the given function is continuous,

$$\therefore k = \lim_{x \rightarrow 0} \frac{\sin x}{x} + \cos x$$

$$\Rightarrow k = 1 + 1 = 2$$

12. **(c)** If an LPP admits two optimal solutions it has an infinite number of optimal solutions

**Explanation:** It is known that the optimal solution of an LPP either exists uniquely, does not exist or exists infinitely. So, If a LPP admits two optimal solution it has an infinite number of optimal solutions



13. (a)  $k > 3$

**Explanation:**  $f(x) = kx^3 - 9x^2 + 9x + 3$

$$f'(x) = 3kx^2 - 18x + 9$$

$$= 3(kx^2 - 6x + 3)$$

Given:  $f(x)$  is monotonically increasing in every interval.

$$\Rightarrow f'(x) > 0$$

$$\Rightarrow 3(kx^2 - 6x + 3) > 0$$

$$\Rightarrow (kx^2 - 6x + 3) > 0$$

$$\Rightarrow K > 0 \text{ and } (-6)^2 - 4(k)(3) < 0 \text{ [} \because ax^2 + bx + c > 0 \text{ and D is } c < 0 \text{]}$$

$$\Rightarrow k > 0 \text{ and } (-6)^2 - 4(k)(3) < 0$$

$$\Rightarrow k > 0 \text{ and } 36 - 12k < 0$$

$$\Rightarrow k > 0 \text{ and } 12k > 36$$

$$\Rightarrow k > 0 \text{ and } k > 3$$

$$\Rightarrow k > 3$$

14. (a) none of these

**Explanation:** Given that  $f(x) = \begin{cases} \frac{1}{1+e^{\frac{1}{x}}}, & x \neq 0 \\ 0, & x = 0 \end{cases}$

Checking continuity at  $x = 0$ ,

$$\text{LHL: } \lim_{x \rightarrow 0^-} \frac{1}{1+e^{\frac{1}{x}}} = 1$$

$$\text{But } f(x = 0) = 0$$

Hence, function is neither continuous nor differentiable at  $x = 0$

15. (b) continuous everywhere but not differentiable at  $x = 0$

**Explanation:** Let  $u(x) = |x|$  and  $v(x) = e^x$

$$\therefore f(x) = v \circ f(x) = v[u(x)]$$

$$= v|x| = e^{|x|}$$

Since,  $u(x)$  and  $v(x)$  are both continuous functions.

So  $f(x)$  is also continuous function but  $u(x) = |x|$  is not differentiable at  $x = 0$ , whereas  $v(x) = e^x$  is differentiable at everywhere.

Hence,  $f(x)$  is continuous everywhere but not differentiable at  $x = 0$ .

16. (d) odd and increasing

**Explanation:** odd and increasing

17. (a) (1, 0)

**Explanation:** We have

$$y = x^2 - 3x + 2$$

Slope of tangent,

$$\frac{dy}{dx} = 2x - 3$$

Tangent perpendicular to this line,

Slope of tangent

$$2x - 3 = -1$$

$$\Rightarrow x = 1$$

$$\text{Now, } y = 1 - 3 + 2$$

$$\Rightarrow y = 0$$

Point is (1, 0)

18. (a)  $\pi$

**Explanation:** The given equation is  $\cos^{-1}(\cos \frac{2\pi}{3}) + \sin^{-1}(\sin \frac{2\pi}{3})$

Let us consider  $\cos^{-1}(\cos(\frac{2\pi}{3}))$  ( $\because$  the principle value of  $\cos$  lies in the range  $[0, \pi]$  and since  $\frac{2\pi}{3} \in [0, \pi]$ )

$$\Rightarrow \cos^{-1}(\cos(\frac{2\pi}{3})) = \frac{2\pi}{3}$$



Also, consider  $\sin^{-1}(\sin(\frac{2\pi}{3}))$

Since here the principle value of sine lies in range  $[-\frac{\pi}{2}, \frac{\pi}{2}]$  and since  $\frac{2\pi}{3} \notin [-\frac{\pi}{2}, \frac{\pi}{2}]$

$$\Rightarrow \sin^{-1}(\sin(\frac{2\pi}{3})) = \sin^{-1}(\sin(\pi - \frac{\pi}{3}))$$

$$= \sin^{-1}(\sin(\frac{\pi}{3}))$$

$$= \frac{\pi}{3}$$

Therefore,

$$\cos^{-1}(\cos(\frac{2\pi}{3})) + \sin^{-1}(\sin(\frac{2\pi}{3})) = \frac{2\pi}{3} + \frac{\pi}{3}$$

$$= \frac{3\pi}{3}$$

$$= \pi.$$

Which is the required solution.

19. (d) 1/8

**Explanation:** If  $f(x)$  is continuous at  $x = \frac{\pi}{2}$  then

$$\lim_{x \rightarrow \frac{\pi}{2}} f(x) = f\left(\frac{\pi}{2}\right)$$

$$\lim_{x \rightarrow \frac{\pi}{2}} \frac{1 - \sin x}{(\pi - 2x)^2} = f\left(\frac{\pi}{2}\right) \dots (i)$$

Now let's suppose

$(\frac{\pi}{2} - x) = t$ , then limit becomes

$$\lim_{t \rightarrow 0} \left[ \frac{1 - \sin(\frac{\pi}{2} - t)}{(2t)^2} \right] = f\left(\frac{\pi}{2}\right) \quad [\text{from equation (i)}]$$

$$\Rightarrow \lim_{t \rightarrow 0} \left[ \frac{1 - \cos t}{4t^2} \right] = f\left(\frac{\pi}{2}\right)$$

$$\Rightarrow \frac{1}{4} \lim_{t \rightarrow 0} \left[ \frac{2 \sin^2\left(\frac{t}{2}\right)}{t^2} \right] = f\left(\frac{\pi}{2}\right)$$

$$\Rightarrow \frac{1}{4} \lim_{t \rightarrow 0} \left[ \frac{\frac{2}{4} \sin^2\left(\frac{t}{2}\right)}{\frac{t^2}{4}} \right] = f\left(\frac{\pi}{2}\right)$$

$$\Rightarrow \frac{1}{8} \lim_{t \rightarrow 0} \left[ \frac{\sin^2\left(\frac{t}{2}\right)}{\frac{t^2}{4}} \right] = f\left(\frac{\pi}{2}\right)$$

$$\Rightarrow \frac{1}{8} \lim_{t \rightarrow 0} \left[ \frac{\sin\left(\frac{t}{2}\right)}{\frac{t}{2}} \right]^2 = f\left(\frac{\pi}{2}\right)$$

$$\Rightarrow f\left(\frac{\pi}{2}\right) = \lambda = \frac{1}{8}$$

20. (d) parallel

**Explanation:** Given  $y = x^3 + 3x$

$$\Rightarrow \frac{dy}{dx} = 3x^2 + 3$$

Slope of tangent at  $x = 1 = 6$  and

Slope of tangent at  $x = -1 = 6$

Hence, the two tangents are parallel.

## Section B

21. (c) one-one and into

**Explanation:**  $f(x) = x^2 + x + 1$

One-one function

Let  $p, q$  be two arbitrary elements in  $N$

Then,  $f(p) = f(q)$

$$\Rightarrow p^2 + p + 1 = q^2 + q + 1$$

$$\Rightarrow p^2 - q^2 + p - q = 0$$

$$\Rightarrow (p - q)(p + q + 1) = 0$$



$$\Rightarrow p = q, p + q + 1 \neq 0 (\because p, q \in \mathbb{N})$$

When  $f(p) = f(q)$ ,  $p = q$

thus,  $f(x)$  is one-one function.

Onto function

For  $x = 1$ ,  $f(x)$  assumes value 3.

As,  $f(x)$  cannot assume value less than 3, for  $x \in \mathbb{N}$

Thus,  $f(x)$  is not onto function. It is into function.

22. (a) Minimum at  $x = \frac{\pi}{2}$

**Explanation:**  $f(x) = 1 + 2 \sin x + 3 \cos^2 x$

$$\Rightarrow f'(x) = 2 \cos x - 6 \cos x \sin x$$

$$\Rightarrow f'(x) = 2 \cos x - 3 \sin 2x$$

to find minima or maxima of the function

$$2 \cos x - 6 \cos x \sin x = 0$$

$$2 \cos x (1 - 3 \sin x) = 0$$

$$\Rightarrow \cos x = 0 \text{ or } \sin x = \frac{1}{3}$$

$$x = \frac{\pi}{2} \text{ or } x = \sin^{-1}\left(\frac{1}{3}\right)$$

$$f''(x) = -2 \sin x - 6 \cos 2x$$

$$f''\left(\frac{\pi}{2}\right) = 4 > 0$$

$\Rightarrow x = \frac{\pi}{2}$  is a local minima.

$$f''\left(\sin^{-1}\left(\frac{1}{3}\right)\right) = -\left(\frac{2}{3} + 4\sqrt{2}\right) < 0$$

function has maxima at  $x = \sin^{-1}\left(\frac{1}{3}\right)$

23. (d) 43

**Explanation:**

Corner points	$Z = 5x + 7y$
O(0,0)	0
B(3,4)	43
A(7,0)	35
C(0,2)	14

Hence the maximum value is 43

24. (d) 75

**Explanation:**  $f(x) = x^2 + \frac{250}{x}$

$$\Rightarrow f'(x) = 2x - \frac{250}{x^2}$$

For the local minima or maxima we must have

$$f'(x) = 0$$

$$2x - \frac{250}{x^2} = 0$$

$$\Rightarrow x = 5$$

$$f''(x) = 2 + \frac{500}{x^3}$$

$$f''(5) = 2 + \frac{500}{125} > 0$$

function has minima at  $x = 5$

$$f(5) = 75$$

25. (d)  $f$  is everywhere continuous but not differentiable at  $x = n\pi, n \in \mathbb{Z}$

**Explanation:** We have,  $f(x) = |\sin x|$

Let  $f(x) = \text{vou}(x) = v[u(x)]$  [where,  $u(x) = \sin x$  and  $v(x) = |x|$ ]

$$= v(\sin x) = |\sin x|$$

Where,  $u(x)$  and  $v(x)$  are both continuous.

Hence,  $f(x) = \text{vou}(x)$  is also a continuous function but  $v(x)$  is not differentiable at  $x = 0$



So,  $f(x)$  is not differentiable where  $\sin x = 0 \Rightarrow x = n\pi, n \in \mathbb{Z}$

Hence,  $f(x)$  is continuous everywhere but not differentiable at  $x = n\pi, n \in \mathbb{Z}$

26. (d)  $\frac{\pi}{6}$

**Explanation:** Let the principle value be given by  $x$

Now, let  $x = \operatorname{cosec}^{-1}(2)$

$$\Rightarrow \operatorname{cosec} x = 2$$

$$\Rightarrow \operatorname{cosec} x = \operatorname{cosec} \left( \frac{\pi}{6} \right) \left( \because \cos \left( \frac{\pi}{6} \right) = 2 \right)$$

$$\Rightarrow x = \frac{\pi}{6}$$

27. (d) one - one onto

**Explanation:** Injectivity: Let  $x_1, x_2 \in \mathbb{R}$  such that  $f(x_1) = f(x_2)$ . Then,  $f(x_1) = f(x_2) \Rightarrow 3x_1 = 3x_2$   
 $\Rightarrow x_1 = x_2$ . So,  $f: \mathbb{R} \rightarrow \mathbb{R}$  is one - one.

Surjectivity: Let  $y \in \mathbb{R}$ , Then  $f(x) = y \Rightarrow 3x = y \Rightarrow x = \frac{y}{3}$ , Clearly,  $\frac{y}{3} \in \mathbb{R}$  for any  $y \in \mathbb{R}$  such that  
 $f \left( \frac{y}{3} \right) = 3 \left( \frac{y}{3} \right) = y$ . So, Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  is onto.

28. (a) 0

**Explanation:**  $\cos^{-1} \left( \cos \frac{5\pi}{3} \right) + \sin^{-1} \left( \sin \frac{5\pi}{3} \right)$   
 $= \cos^{-1} \left( \cos \left( 2\pi - \frac{\pi}{3} \right) \right) + \sin^{-1} \left( \sin \left( 2\pi - \frac{\pi}{3} \right) \right)$   
 $= \cos^{-1} \left( \cos \left( \frac{\pi}{3} \right) \right) + \sin^{-1} \left( -\sin \left( \frac{\pi}{3} \right) \right)$   
 $= \cos^{-1} \left( \cos \left( \frac{\pi}{3} \right) \right) - \sin^{-1} \left( \sin \left( \frac{\pi}{3} \right) \right)$   
 $= \frac{\pi}{3} - \frac{\pi}{3}$   
 $= 0$

29. (d)  $\left( 0, \frac{1}{e} \right)$

**Explanation:**  $\left( 0, \frac{1}{e} \right)$

Let  $y = x^x$

$$\Rightarrow \log(y) = x \log x$$

$$\Rightarrow \frac{1}{y} \times \frac{dy}{dx} = 1 + \log x$$

$$\Rightarrow \frac{dy}{dx} = x^x (1 + \log x)$$

Since the function is decreasing,

$$\Rightarrow x^x x (1 + \log x) < 0$$

$$\Rightarrow 1 + \log x < 0$$

$$\Rightarrow \log x < -1$$

$$\Rightarrow x < \frac{1}{e}$$

Therefore, function is decreasing on  $\left( 0, \frac{1}{e} \right)$

30. (c) Determinant is a number associated to a square matrix.

**Explanation:** The determinant is an operation that we perform on arranged numbers. A square matrix is a set of arranged numbers. We perform some operations on a matrix and we get a value that value is called as a determinant of that matrix hence a determinant is a number associated to a square matrix.

31. (c)  $n^2 y$

**Explanation:**  $y^{1/n} + y^{-1/n} = 2x$

Differentiating both sides we get

$$\frac{y_1}{n} \left( y^{\frac{1}{n}-1} - y^{\frac{-1}{n}-1} \right) = 2$$

$$\Rightarrow y_1 \left( y^{\frac{1}{n}} - y^{\frac{-1}{n}} \right) = 2ny$$

Again differentiating both sides we get

$$y_2 \left( y^{\frac{1}{n}} - y^{\frac{-1}{n}} \right) + \frac{y_1}{n} \left( y^{\frac{1}{n}-1} + y^{\frac{-1}{n}-1} \right) = 2ny_1$$

$$\Rightarrow ny_2 \left( y^{\frac{1}{n}} - y^{\frac{-1}{n}} \right) + \frac{y_1^2}{y} \left( y^{\frac{1}{n}} + y^{\frac{-1}{n}} \right) = 2n^2 y_1$$

$$\Rightarrow ny_2 \left( y^{\frac{1}{n}} - y^{\frac{-1}{n}} \right) + 2xy_1^2 = 2n^2 y_1$$



$$\begin{aligned} \Rightarrow nyy_2 \frac{2ny}{y_1} + 2xy_1^2 &= 2n^2yy_1 \\ \Rightarrow \frac{n^2y^2y_2}{y_1^2} + xy_1 &= n^2y \\ \Rightarrow y_2 \frac{\left(y^{\frac{1}{n}} - y^{\frac{-1}{n}}\right)^2}{4} + xy_1 &= n^2y \\ \Rightarrow y_2 \frac{\left(y^{\frac{1}{n}} + y^{\frac{-1}{n}}\right)^2 - 4}{4} + xy_1 &= n^2y \\ \Rightarrow y_2 \frac{4x^2 - 4}{4} + xy_1 &= n^2y \\ \Rightarrow (x^2 - 1)y_2 + xy_1 &= n^2y \end{aligned}$$

32. (c) 0

**Explanation:** Since, f is continuous at  $x = \frac{\pi}{2}$

$$\therefore f\left(\frac{\pi}{2}\right) = \lim_{x \rightarrow \frac{\pi}{2}} \frac{\sin(\cos x - \cos x)}{(\pi - 2x)^2}$$

$$\text{i.e. } k = \lim_{x \rightarrow \frac{\pi}{2}} \frac{\sin(\cos x - \cos x)}{(\pi - 2x)^2}$$

Let  $x = \frac{\pi}{2} - h$ ,

$$\Rightarrow k = \lim_{h \rightarrow 0} \frac{\sin(\cos(\frac{\pi}{2} - h) - \cos(\frac{\pi}{2} - h))}{\left(\pi - 2\left(\frac{\pi}{2} - h\right)\right)^2}$$

$$= \lim_{h \rightarrow 0} \frac{\sin(\sin h) - \sin h}{4h^2}$$

Using  $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \dots$

$$\Rightarrow k = \lim_{h \rightarrow 0} \frac{(\sin h - \frac{\sin^3 h}{3!} + \frac{\sin^5 h}{5!} \dots) - \sin h}{4h^2}$$

$$= \lim_{h \rightarrow 0} \left( \frac{-\sin^3 h}{3! \times 4h^2} + \frac{\sin^5 h}{5! \times 4h^2} \dots \right)$$

$$= 0$$

$$\therefore \lim_{x \rightarrow \frac{\pi}{2}} f(x) = 0 = k$$

$$\Rightarrow k = 0$$

33. (d) monotonically decreasing

**Explanation:** monotonically decreasing

34. (c)  $\frac{\pi}{4}$

$$\text{Explanation: } \tan^{-1} \frac{1}{7} + 2 \tan^{-1} \frac{3}{4} \Rightarrow \tan^{-1} \frac{1}{7} + \tan^{-1} \frac{2 \cdot \frac{3}{4}}{1 - \left(\frac{3}{4}\right)^2}$$

$$= \tan^{-1} \frac{1}{7} + 2 \tan^{-1} \frac{3}{4} \Rightarrow \tan^{-1} \frac{\frac{1}{7} + \frac{3}{4}}{1 - \frac{1}{7} \cdot \frac{3}{4}} \Rightarrow \tan^{-1}(1) = \frac{\pi}{4}$$

35. (c)  $\frac{1}{2}$

**Explanation:** We know that  $A \times A^{-1} = I$

$$\begin{pmatrix} 2x & 0 \\ x & x \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 2x \times 1 + 0 \times (-1) & 2x \times 0 + 0 \times 2 \\ x \times 1 + x \times (-1) & x \times 0 + x \times 2x \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 2x & 0 \\ 0 & 2x \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

To satisfy the above condition  $2x = 1$

$$x = \frac{1}{2}$$

36. (b) at any vertex of feasible region

**Explanation:** In linear programming problem we substitute the coordinates of vertices of feasible region

in the objective function and then we obtain the maximum or minimum value. Therefore, the value of objective function is maximum under linear constraints at any vertex of feasible region.

37. (b) none of these

**Explanation:** We have,  $A = \frac{1}{3} \begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & -2 \\ x & 2 & y \end{bmatrix}$

$$\Rightarrow A^T = \frac{1}{3} \begin{bmatrix} 1 & 2 & x \\ 1 & 1 & 2 \\ 2 & -2 & y \end{bmatrix}$$

Now,  $A^T A = I$

$$\Rightarrow \begin{bmatrix} x^2 + 5 & 2x + 3 & xy - 2 \\ 3 + 2x & 6 & 2y \\ xy - 6 & 2y & y^2 + 8 \end{bmatrix} = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

The corresponding elements of two equal matrices are not equal.

Thus, the matrix A is not orthogonal.

38. (b)  $\frac{4}{3}$

**Explanation:**  $f(x) = \frac{1}{4x^2 + 2x + 1}$

$$\Rightarrow f'(x) = 8x + 2$$

For local minima or maxima we have

$$f'(x) = 8x + 2 = 0$$

$$\Rightarrow x = \frac{-1}{4}$$

$$f''(x) = 8 > 0$$

$$\Rightarrow \text{function has maxima at } x = \frac{-1}{4}$$

$$f\left(\frac{-1}{4}\right) = \frac{4}{3}$$

39. (c)  $m^2y$

**Explanation:**  $y = ae^{mx} + be^{-mx} \Rightarrow y_1 = ame^{mx} + (-m)be^{-mx} \Rightarrow y_2 = am^2e^{mx} + (m^2)be^{-mx}$   
 $\Rightarrow y_2 = m^2(ae^{mx} + be^{-mx}) \Rightarrow y_2 = m^2y$

40. (d) {1, 3}, {2, 4, 5}, {6}

**Explanation:** Conditions for the partition sub-sets to be an equivalence relation:

The partition sub-sets must be disjoint i.e. there is no common elements between them

Their union must be equal to the main set (super-set)

Here, the set  $A = \{1, 2, 3, 4, 5, 6\}$ , the partition sub-sets  $\{1, 3\}, \{2, 4, 5\}, \{6\}$  are pairwise disjoint and their union i.e.  $\{1, 3\} \cup \{2, 4, 5\} \cup \{6\} = \{1, 2, 3, 4, 5, 6\} = A$ , which is the condition for the partition sub-sets to be an equivalence relation of the set A.

### Section C

41. (c)  $\frac{1}{2\sqrt{2}}$

**Explanation:**  $\sin\left(\frac{1}{4}\sin^{-1}\frac{\sqrt{63}}{8}\right)$

$$\text{Let, } \sin^{-1}\frac{\sqrt{63}}{8} = x$$

$$\sin x = \frac{\sqrt{63}}{8}$$

$$\cos x = \sqrt{1 - \sin^2 x}$$

$$\cos x = \sqrt{1 - \frac{63}{64}}$$

$$\cos x = \frac{1}{8}$$

Consider,

$$\sin\left(\frac{1}{4}\sin^{-1}\frac{\sqrt{63}}{8}\right)$$

$$= \sin\left(\frac{1}{4}x\right)$$





$$\begin{aligned}
&= \sqrt{\frac{1 - \cos \frac{x}{2}}{2}} \quad \left( \because \sin x = \frac{1 - \cos 2x}{2} \right) \\
&= \sqrt{\frac{1 - \sqrt{\frac{1 + \cos x}{2}}}{2}} \quad \left( \because \cos x = \frac{1 + \cos 2x}{2} \right) \\
&= \sqrt{\frac{1 - \sqrt{\frac{1 + \frac{1}{8}}{2}}}{2}} \\
&= \sqrt{\frac{1 - \frac{3}{4}}{2}} \\
&= \sqrt{\frac{1}{8}} \\
&= \frac{1}{2\sqrt{2}}
\end{aligned}$$

42. (c) 1700

**Explanation:** Here, Maximize  $Z = 50x + 60y$ , subject to constraints  $x + 2y \leq 50$ ,  $x + y \geq 30$ ,  $x, y \geq 0$ .

Corner points	$Z = 50x + 60y$
P(50, 0)	2500
Q(0, 30)	1800
R(10, 20)	1700

Hence the minimum value is 1700

43. (a) 1

**Explanation:**  $f'(6) = \lim_{h \rightarrow 0} \frac{f(6+h) - f(6)}{h} = \lim_{h \rightarrow 0} \frac{f(6+h) - f(6+0)}{h}$

$$\begin{aligned}
&= \lim_{h \rightarrow 0} \frac{f(6) + f(h) - \{f(6) + f(0)\}}{h} \\
&= \lim_{h \rightarrow 0} \frac{f(h) - f(0)}{h} = f'(0) = 1
\end{aligned}$$

44. (c)  $|A| + |A'| \neq 0$

**Explanation:** Because, the determinant of a matrix and its transpose are always equal that is  $|A| = |A'|$

45. (b) Transitive and symmetric

**Explanation:** The relation  $\{ \} \subset A \times A$  on  $A$  is surely not reflexive. However, neither symmetry nor transitivity is contradicted. So  $\{ \}$  is a transitive and symmetric relation on  $A$ .

46. (c) ₹32,000

**Explanation:** ₹32,000

47. (d) ₹46,000

**Explanation:** ₹46,000

48. (d) ₹53,000

**Explanation:** ₹53,000

49. (d) ₹31,000

**Explanation:** ₹31,000

50. (c) (₹15,000, ₹17,000)

**Explanation:** (₹15,000, ₹17,000)

